ANALYSIS OF AN INVENTORY MANAGEMENT MODEL WITH DISCOUNTS

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Abstract:
This article deals with the theory of inventory management and sets it in the context of other methods of operational research, and it also emphasizes the importance of this theory for strategic decisions in enterprises and organizations. Next, based on a Master's Thesis 'Inventory Optimization of a Chosen Enterprise' by the author, this article deals with the discount model, which is analysed from the theoretical point of view with an emphasis on the variable part of storage costs. The respective mathematical formulas are also derived to calculate the total costs. Moreover, the mathematical structure of the discount model is compared with that of the EOQ model, which was the first inventory management model proposed as early as 1915. Various examples are given to illustrate the discount model. Finally, the article discusses the advantages and disadvantages of the discount model and formulates recommendations.

Introduction

Inventory management is a developing branch of a large part of operational research, which can be useful in entrepreneurship with inventories, namely, in e-shops, manufacturing industry or catering. In operational research, the theory of inventory management offers many possibilities for efficiently controlling inventory, processes, and profit. These facts have been known for a long time; see, e.g., Silver (1981). Before making a strategical decision in entrepreneurship, it is important to have good and deep knowledge of the inventory. Further information can be found in Muckstadt and Sapra (2010); see also Shah and Mittal (2020).

The aim of this article is to discuss the model with discounts. The model considers the inventory (goods, products, etc.) storage costs when the inventories can be bought with discounts; see Chung et al. (2018). The model is compared with the EOQ model, which is a basic model in the theory of inventory management, see Choi (2014) and Schwarz (2008); see also Muckstadt and Sapra (2010). The formulas for the calculations for the basic quantities in both models are compared.

Inventory binds the capital of the company, and this is why its analysis is necessary for strategic inventory management in order to minimize the total costs, which also include the supply costs the storage costs, and costs due to inventory insufficiency. In general economics, the inventory management is also related with the investment; see, e.g., Thompson (1975) or Harris (1997).

The discount model and the EOQ model are deterministic, which means that the demand for the inventory is known exactly in advance. The stochastic models better reflect the reality because the demand is understood as a random variable, which means that it is not known in advance. In stochastic models, the demand expected value is considered in both the cases of the discrete and continuous demand. See Lukáš (2012) and Choi (2014).

There is an example at the end of this article to illustrate several variants of the model with discounts, and then the results of the variants are compared. In this work, operational research is used.

1. The inventory management model with discount

This model is characterised by Jablonský (2007). As was mentioned in the introduction, this model is deterministic, and it is based on the EOQ model. The supplier offers discounts to the buyer depending on the amount of goods supplied. The larger the amount, the larger the discount. The amount of goods supplied is categorized. There are k discount categories indexed by $i = 1, 2, \dots, k$. The i-th discount category is determined by the interval $\left(\bar{q}^{i-1}, \bar{q}^{i}\right]$ of the amount of goods ordered for $i = 1, 2, \dots, k$, where $0 = \bar{q}^0 < \bar{q}^1 < \bar{q}^2 < \cdots < \bar{q}^{k-1} < \bar{q}^k = +\infty$ are the boundary points of the discount categories.

Let $\bar{c}^1 > \bar{c}^2 > \cdots > \bar{c}^k$ be the unit purchase costs of goods such that $\bar{c}^i$ is the unit purchase price for goods if the ordered quantity q belongs to the i-th discount category, i.e. $q \in \left(\bar{q}^{i-1}, \bar{q}^{i}\right]$, for $i = 1, 2, \dots, k$. The unit purchase price costs c^q, when the ordered quantity is q, is calculated by using the next formula:

$$c_1^q = \begin{cases} \bar{c}_1^1, & \text{if } 0 < q \le \bar{q}^1, \\ \bar{c}_1^2, & \text{if } \bar{q}^1 < q \le \bar{q}^2, \\ \dots & \dots \\ \bar{c}_1^{k-1} & \text{if } \bar{q}^{k-2} < q \le \bar{q}^{k-1}, \\ \bar{c}_1^k & \text{if } \bar{q}^{k-1} < q. \end{cases} \qquad (*)$$

For example, consider $k = 6$ discount categories, with boundary points $0 = \bar{q}^0 < 100 = \bar{q}^1 < 200 = \bar{q}^2 < 300 = \bar{q}^3 < 400 = \bar{q}^4 < 500 = \bar{q}^5 < +\infty = \bar{q}^6$.

The discount category is then given by the particular amount of the goods supplied, that is, from 0 to 100 is the first discount category with the unit price, e.g. $\bar{c}^1 = 1000$ Kč/unit, from 100 to 200 is the second discount category with the unit price, e.g. $\bar{c}^2 = 900$ Kč/unit, from 200 to 300 is the third discount category with the unit price, e.g. $\bar{c}^3 = 800$ Kč/unit, etc., above 500 is the sixth discount category with the unit price, e.g. $\bar{c}^6 = 500$ Kč/unit.

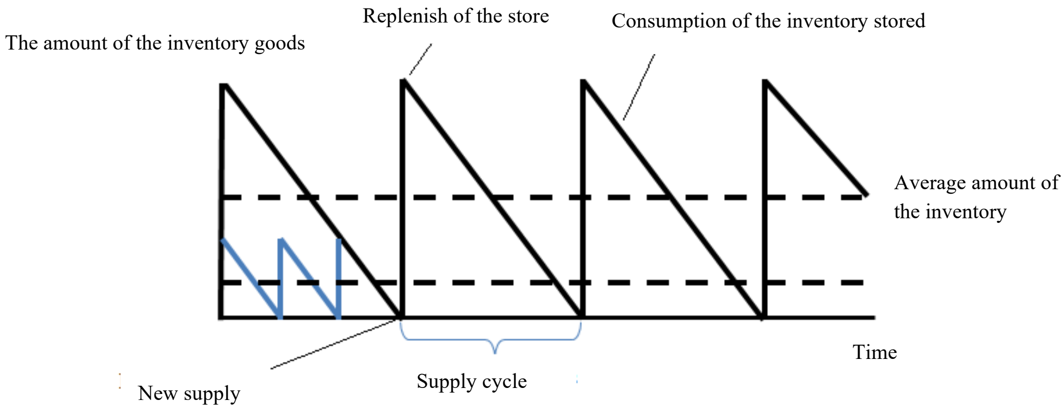
If the ordered quantity is, e.g., $q = 150$, then $q \in (\bar{q}^1, \bar{q}^2] = (100, 200]$, which means that q belongs to the category no. 2.

Analogously, if $\bar{c}_1^1 \ge \bar{c}_1^2 \ge \cdots \ge \bar{c}_1^k$ are the unit storage costs such that $\bar{c}_1^i$ is the unit storage costs for goods, if the ordered quantity q belongs to the i-th discount category, i.e. $q \in \left(\bar{q}^{i-1}, \bar{q}^{i}\right]$, for $i = 1, 2, \dots, k$. The unit storage costs c^q, when the ordered quantity is q, is calculated by using a formula analogous to $(*)$.

Unit storage costs are often given relative to the purchase price. Jablonský (2007) gives that the unit storage costs decrease as the amount of inventory stored increases. However, the author of

this article discusses whether unit storage costs should not be constant. Unit storage costs include, e.g., the rental of the storage facility or manipulation with the stored items.
The following Figure 1 presents a chart of the EOQ model, that is, the progress of amount of the inventory stored over time, which can also be understood as a chart of the discount model because the progress of the amount of the goods stored is the same in both models.

Figure 1: *A chart of the EOQ model and the discount model.*



Source: Jablonský (2007, p. 212), own editing. Taken from Heczková (2021)

The following Table 1 presents the formulas used in the model with discounts and those used in the EOQ model for comparison. In the EOQ model, there are two types of costs considered: storage costs and the purchase costs. Total costs are lower in the EOQ model because the stored capital bound in the inventory is not considered. However, the total costs in the model with discounts also include the third term $c^q \cdot Q$, where c^q are the unit purchase costs of the goods according to the respective discount category and Q is the demand per year in natural units, i.e., the capital bound in the inventory stored.

Table 1: *The formulas used in the EOQ model and in model with discounts*

	Model EOQ	Model with discounts
Total costs, N	$N(q) = c_1 \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q}$ $= \sqrt{2Qc_1c_2}$	$N = c_1^q \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q} + c^q \cdot Q$.
Variable (storage) costs, N_v	$N_v = c_1 \cdot \frac{q}{2}$	$N_v = c_1^q \cdot \frac{q}{2}$
Fixed costs (purchase costs), N_f	$N_f = c_2 \cdot \frac{Q}{q}$	$Nf = c_2 \cdot \frac{Q}{q}$
The third term in the discount model, i.e., the capital bound in the inventory stored, i.e., the alternative costs.		$c^q \cdot Q$
The amount of a single supply $q_{opt} = q^*$	$q^* = \sqrt{\frac{2 \cdot Q \cdot c_2}{c_1}}$	$q_i^* = \sqrt{\frac{2 \cdot Q \cdot c_2}{c_1^q}}$ for $i = 1, 2, \ldots, k$

Source: own work according to Jablonský (2007) and Heczková (2021)

2. An example of inventory management: of iodine-bromine products using the discount model

This example is taken from Heczková (2021) and modified. An enterprise purchases idodine bromine (IBR) products (salt 1kg) from a supplier. Table 2 summarizes the input data for the subsequent calculations.

Table 2: *The input data for the subsequent calculations*

Input data	**IBR salt 1 kg**
Purchase price	109 Kč
Selling price	199 Kč
Margin (sale - purchase price), and also inventory unit price c_3 due to the insufficiency of the inventory	90 Kč
The number of the supply cycles per year	8× per year
The amount of the supply q	120
The demand Q per year	8·120 = 960
The unit storage costs c_1	2 % of the total purchase price of the IBR salt per year = 0,02·109·8·120 = 2092,80 Kč/piece
The unit purchase costs c_2	50 Kč
Time from order to the arrival of the supply to the store d	2 days = 2/360 year = 0,005555556 year

Source: Heczková (2021)

If the enterprise decides to order quantity of 500 or more pieces of the IBR salt, then the discount is 15% of the given purchase price. The calculated data are presented in Table 3.

Table 3: *Data for the calculations of the IBR products by using the discount model*

The amount of the order	Discount [%]	Unit price c^q	Unit storage costs c_1^q
0-500	0	109 Kč	2 % of the purchase price of a piece of IBR salt: 2 % of 109 Kč = 2,18 Kč/piece
500 or more	15 %	109 – 16,35 = 92,65 Kč	Remain the same, 2,18 Kč/piece
500 or more	15 %	109 – 16,35 = 92,65	2 % of the purchase price of a piece of IBR salt: 2 % of 92,65 Kč = 1,853 Kč/piece

Source: Own work according to Jablonský (2007, p. 226). Taken from Heczková (2021), own editing.

Calculation of the optimal amount of a single supply and of the total costs based on the data given in Tables 2 and 3 for the first discount category, that is, from 0 to 500 pieces, according to Heczková (2021):

$$q_1^* = \sqrt{\frac{2 \cdot Q \cdot c_2}{c_1^q}} = \sqrt{\frac{2 \cdot 960 \cdot 50}{2{,}18}} = \sqrt{\frac{96000}{2{,}18}} = \sqrt{44036{,}69725} = 209{,}84923 \text{ pieces}$$

Total costs:

$$N = c_1^q \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q} + c^q \cdot Q = 2{,}18 \cdot \frac{209{,}84923}{2} + 50 \cdot \frac{960}{209{,}84923} + 109 \cdot 960 = \\ = 105097{,}47131 \text{ Kč}$$

Calculation of the optimal amount of a single supply and of the total costs for the second discount category, that is, from 500 or more pieces, according to Heczková (2021):

$$q_2^* = \sqrt{\frac{2 \cdot Q \cdot c_2}{c_1^q}} = \sqrt{\frac{2 \cdot 960 \cdot 50}{2{,}18}} = \sqrt{\frac{96000}{2{,}18}} = \sqrt{44036{,}69725} = 209{,}84923 \text{ ks}$$

Since the optimal amount of a single supply $q_2^* = 209{,}84923$ pieces is not in the second discount category (500 or more pieces), it is necessary to increase the amount to the lower limit of this second category:

$$q_2^* \coloneqq 500$$

Total costs:

$$N = c_1^q \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q} + c^q \cdot Q = 2{,}18 \cdot \frac{500}{2} + 50 \cdot \frac{960}{500} + 92{,}65 \cdot 960 = \\ = 89.585 \text{ Kč}$$

Finally, consider variable unit storage costs, that is, 2% of the discounted purchase price of a piece of IBR salt, i.e., 1,853 Kč/piece:

$$q_1^* = \sqrt{\frac{2 \cdot Q \cdot c_2}{c_1^q}} = \sqrt{\frac{2 \cdot 960 \cdot 50}{1,853}} = \sqrt{\frac{96000}{1,853}} = \sqrt{51807,87911} = 227,6134423 \text{ pieces}$$

As above, since the optimal amount of a single supply $q_2^* = 227,6134$ pieces is not in the second discount category (500 or more pieces), it is necessary to increase the amount to the lower limit of this second category:

$$q_2^* \coloneqq 500$$

Total costs:

$$N = c_1^q \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q} + c^q \cdot Q = 1,853 \cdot \frac{500}{2} + 50 \cdot \frac{960}{500} + 92,65 \cdot 960 =$$
$$= 463,2500000 + 96 + 88944 = 89.503,25\ K\check{c}.$$

It follows that it is beneficial to utilize the discount when taking $q = 500$ pieces because the total costs are lower than in the case without using the discounts.

The calculation of the total costs using the EOQ model based on the data given in Tables 2 and 3

$$N(q) = c_1 \cdot \frac{q}{2} + c_2 \cdot \frac{Q}{q} = 2,18 \cdot \frac{500}{2} + 50 \cdot \frac{960}{500} = 545 + 96 = 641\ K\check{c}$$

It follows that using the EOQ model the total costs are about 641 Kč and by using the model with discounts the total costs are about 89.585 Kč in the first variant of the example and 89.503,25 Kč in the second variant of the example.

Conclusion

In practice, the use of the inventory management model with discounts is limited. The article compared the structure of the EOQ model and that of the model with discounts. The total costs calculated using the EOQ model are always less than the total costs calculated by using the model with discounts. This is because the model with discounts contains the term expressing the alternative costs, i.e., the capital bound in the inventory stored. Considering the storage costs, the author concludes that the unit storage costs should remain the same irrespectively of the amount of inventory stored. This is because the labour of the store worker to handle a single item of the inventory is the same irrespectively of the amount of the inventory items stored, which is a conclusion different from the opinion of Jablonský (2007).

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